Rough set theory and concept lattices to solve fuzzy relation equations*

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Property-oriented concept lattices and object-oriented concept lattices [5, 11, 14] arose as a generalization of rough set theory (RST) considering the philosophy of formal concept analysis (FCA) [13]. These frameworks were introduced in the multi-adjoint fuzzy setting [6] in the paper [17]. Specifically, the lower and upper approximation operators are generalized to be applied to a quantitative relationship between the objects and attributes (informative table) instead of its discernibility relation [8, 17]. The new (modal) operators are called necessity and possibility operators, respectively.

Fuzzy relation equations (FRE) are associated with the composition of fuzzy relations and is an appropriate tool for handling and modeling non-probabilistic forms of uncertainty. From their introduction in the eighties by Elie Sanchez [19], FRE have received great interest from their introduction due to the wide range of their applications in different areas of real-world, such as decision making [7], bipolarity [12], optimization [2] and image processing [1].

A relevant achievement was to characterize the solvability of FRE with the computation of concepts in a property-oriented concept lattice or an object-oriented concept lattice [9, 10]. Given a FRE, depending on whether the unknown relation is to the left side or to the right side, we have determined an associated property-oriented context or an object-oriented context, respectively, whose concepts provide solvable FREs related to the original one [9].

This relationship has allowed diverse advances in FRE. For example, it has been possible to introduce three different mechanisms of approximating unsolvable FRE. Two of them are based on the computation of closest concepts associated with a given fuzzy subset of attributes [7] and the third one is based on the theory of attribute reduction [3, 4, 18] developed in the mentioned concept lattices frameworks [16, 15]. This last theory has also allow the reduction of equations of the system associated with a given FRE [15], which is a novel procedure that had not been envisaged until now.

This contribution will be focused on these relations and achievements, and will present new challenges.

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